

Single-photon multi-ports router based on the coupled cavity optomechanical system

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ABSTRACT

A scheme of single-photon multi-port router is put forward by coupling two optomechanical cavities with waveguides. It is shown that the coupled two optomechanical cavities can exhibit photon blockade effect, which is generated from interference of three mode interaction. A single-photon travel along the system is calculated. The results show that the single photon can be controlled in the multi-port system because of the radiation pressure, which should be useful for constructing quantum network.

Introduction

Quantum router to combine quantum channels with quantum nodes can create a quantum network so as to distribute quantum information. Recently, many theoretical proposals and experimental demonstrations of a quantum router have been carried out in various systems, i.e., cavity QED system^{1,2}, nanocavity arrays coupling with three-level system^{3,4}, cavity electromechanical system⁵, all-linear-optical scheme⁶⁻⁸ and optomechanical systems⁹.

Photon-blockade phenomenon resulted from nonlinearity allows only one photon existence, and the second photon will be prohibited, which can be used to generate single photon source or to ensure a single photon processing. Cavity optomechanical systems, besides its potential application in detecting gravity waves^{10,11}, in studying quantum-to-classical transitions¹², in performing high precision measurements¹³⁻¹⁶, in entanglement generation^{17,18} and preservation¹⁹ and in processing quantum information^{15,21-24}, are of nonlinearity²⁵⁻³¹. But this nonlinear strength proportional to g^2/ω_m is limited by the condition g (the coupling strength of radiation pressure) less than ω_m (the frequency of the mechanical oscillator), therefore, a lot of effort is devoted to enhance the nonlinearity, for instance, adding atoms³², introducing quantum dot³³, using coupled cavity optomechanical system²⁶. In Ref.³⁴, the authors had put forward a approach where the photons in the two optical modes can be resonantly exchanged by absorbing or emitting a phonon via three-mode mixing so as to generate effective photon blockade not depending on the ratio g^2/ω_m .

In this paper, we put forward a scheme by coupling two cavity optomechanical system. We show that our system can be effective equal to three-mode interaction³⁴ and can exhibit photon blockade. Then we construct four output ports by coupling wave guide to the two-cavity-optomechanical system. Our research show that our system can work as multiple output ports router under the assistant of mechanical mode, which provide a potential application for the cavity optomechanical system in multiple router.

Results

In this part, we introduce the model, illustrate the photon-blockade effect of this two-cavity-optomechanical waveguide coupled system and study the transport of photons of waveguide under photon-blockade condition.

Model and effective interaction

We consider the two optomechanical cavities coupled with hopping coefficient J where the two cavities are driven by two laser beam with frequencies ω_{L_1} and ω_{L_2} separately, and the two optomechanical cavities are side-coupled to the fibres. The configuration of the system is shown in Fig.1a, which is similar with Ref.³⁵ where they utilized the two coupled whispering-gallery-mode (WGM) microtoroids coupled to two tapered fibres to experimentally realize parity-time-symmetric optics, but the mechanical modes are ignored. Taking the mechanical modes into consideration, we write the Hamiltonian as

$$\hat{H} = \hat{H}_{cav} + \hat{H}_{om} + \hat{H}_f \quad (1)$$

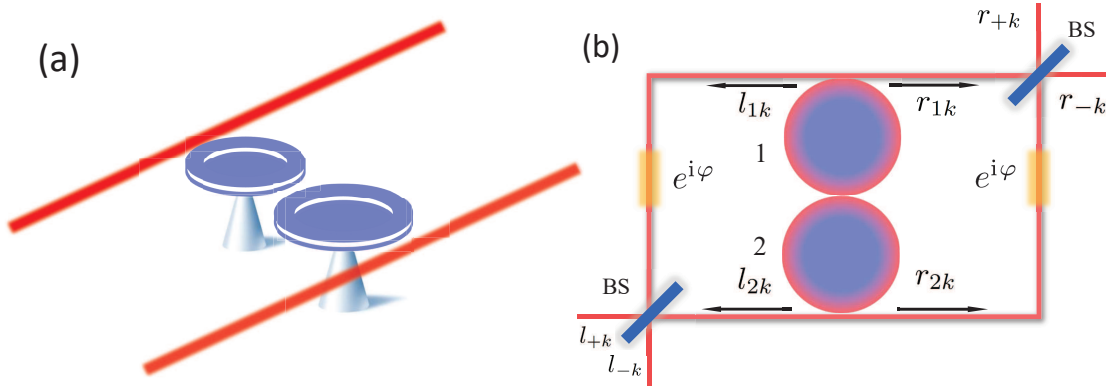


Figure 1. Schematic configuration of the single-photon router. (a) The two toroidal cavities with mechanical modes coupling to waveguide. (b) The four ports router with quasi-mode.

with

$$\begin{aligned}\hat{H}_{cav} &= \omega_1 \hat{a}_1^\dagger \hat{a}_1 + \omega_2 \hat{a}_2^\dagger \hat{a}_2 + J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger) + \sum_{j=1,2} \epsilon_j (\hat{a}_j e^{i\omega_{L_j} t} + \hat{a}_j^\dagger e^{-i\omega_{L_j} t}), \\ \hat{H}_{om} &= \omega_{m1} \hat{b}_1^\dagger \hat{b}_1 + \omega_{m2} \hat{b}_2^\dagger \hat{b}_2 + g \hat{a}_1^\dagger \hat{a}_1 (\hat{b}_1 + \hat{b}_1^\dagger) + g \hat{a}_2^\dagger \hat{a}_2 (\hat{b}_2 + \hat{b}_2^\dagger),\end{aligned}$$

where \hat{H}_{cav} describes the free energy of the cavity, the exchange energy between the two cavities with hopping strength J as well as the two classical driving laser with frequencies ω_{L_1} and ω_{L_2} respectively, where \hat{a}_1 (\hat{a}_2) and \hat{a}_1^\dagger (\hat{a}_2^\dagger) represent the annihilation and creation operators of cavity modes. \hat{H}_{om} represents the energy of the two mechanical oscillators and their coupling with the cavity fields induced by radiation pressure, where the \hat{b}_1 (\hat{b}_2) and \hat{b}_1^\dagger (\hat{b}_2^\dagger) are annihilation and creation operators of mechanical oscillators. The Hamiltonian \hat{H}_f in Eq.(1) can be written as

$$\hat{H}_f = \sum_{O=r,l} \sum_{j=1,2} \int_{-\infty}^{+\infty} dk \left[\omega_k \hat{O}_{jk}^\dagger \hat{O}_{jk} + i\xi (\hat{a}_j^\dagger \hat{O}_{jk} - \hat{a}_j \hat{O}_{jk}^\dagger) \right], \quad (2)$$

which expresses the two cavity fields coupling with the fibres, where \hat{O}_{jk} ($j = 1, 2$; $\hat{O} = r, l$) represents annihilation operators of the fibres. In the frame rotating with $\hat{H}_0 = \omega_{L_1} [\hat{a}_1^\dagger \hat{a}_1 + \sum_{\hat{O}=r,l} \int_{-\infty}^{+\infty} dk \hat{O}_{1k}^\dagger \hat{O}_{1k}] + \omega_{L_2} [\hat{a}_2^\dagger \hat{a}_2 + \sum_{\hat{O}=r,l} \int_{-\infty}^{+\infty} dk \hat{O}_{2k}^\dagger \hat{O}_{2k}]$, we have

$$\begin{aligned}\hat{H}'_{cav} &= \Delta (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) - J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger) + \sum_{j=1,2} \epsilon_j (\hat{a}_j + \hat{a}_j^\dagger), \\ \hat{H}'_{om} &= \omega_{m1} \hat{b}_1^\dagger \hat{b}_1 + \omega_{m2} \hat{b}_2^\dagger \hat{b}_2 + g_1 \hat{a}_1^\dagger \hat{a}_1 (\hat{b}_1 + \hat{b}_1^\dagger) + g_2 \hat{a}_2^\dagger \hat{a}_2 (\hat{b}_2 + \hat{b}_2^\dagger),\end{aligned} \quad (3)$$

where for simplicity we assume $\omega_1 = \omega_2$ and $\omega_{L_1} = \omega_{L_2}$ so that $\Delta = \omega_1 - \omega_{L_1} = \omega_2 - \omega_{L_2}$, and

$$\hat{H}_f = \sum_{O=r,l} \sum_{j=1,2} \int_{-\infty}^{+\infty} dk \left[\Delta_k \hat{O}_{jk}^\dagger \hat{O}_{jk} + i\xi (\hat{a}_j^\dagger \hat{O}_{jk} - \hat{a}_j \hat{O}_{jk}^\dagger) \right], \quad (4)$$

where $\Delta_k = \omega_k - \omega_{L_1} = \omega_k - \omega_{L_2}$. Now, we introduce the operators

$$\hat{a}_\pm = \frac{1}{\sqrt{2}}(\hat{a}_1 \pm \hat{a}_2), \hat{b}_\pm = \frac{1}{\sqrt{2}}(\hat{b}_1 \pm \hat{b}_2),$$

The Hamiltonian $\hat{H}_s = \hat{H}_{cav} + \hat{H}_{om}$ is of the form

$$\begin{aligned}\hat{H}_s &= \Delta_+ \hat{a}_+^\dagger \hat{a}_+ + \Delta_- \hat{a}_-^\dagger \hat{a}_- + \epsilon_- (\hat{a}_-^\dagger + \hat{a}_-) + \epsilon_+ (\hat{a}_+^\dagger + \hat{a}_+) + \omega_m \hat{b}_+^\dagger \hat{b}_+ + \omega_m \hat{b}_-^\dagger \hat{b}_- \\ &\quad + \frac{g}{\sqrt{2}} (\hat{b}_+ + \hat{b}_+^\dagger) (\hat{a}_+^\dagger \hat{a}_+ + \hat{a}_+ \hat{a}_+^\dagger) + \frac{g}{\sqrt{2}} (\hat{b}_- + \hat{b}_-^\dagger) (\hat{a}_-^\dagger \hat{a}_- + \hat{a}_- \hat{a}_-^\dagger),\end{aligned} \quad (5)$$

where we have assume $g_1 = g_2 = g$, $\Delta_\pm = \Delta \mp J$, $\epsilon_\pm = \frac{\epsilon_1 \pm \epsilon_2}{\sqrt{2}}$. For the fibre, we define

$$\hat{r}_{\pm k} = \frac{1}{\sqrt{2}}(\hat{r}_{1k} \pm \hat{r}_{2k}), \hat{l}_{\pm k} = \frac{1}{\sqrt{2}}(\hat{l}_{1k} \pm \hat{l}_{2k}), \hat{d}_{\pm k} = \frac{\hat{r}_{\pm k} + \hat{l}_{\pm k}}{\sqrt{2}}, \hat{c}_{\pm k} = \frac{\hat{r}_{\pm k} - \hat{l}_{\pm k}}{\sqrt{2}}. \quad (6)$$

Thus, \hat{H}_f can be rewritten as

$$\hat{H}_f = \int_0^\infty \Delta_k dk [\hat{d}_{+k}^\dagger \hat{d}_{+k} + \hat{d}_{-k}^\dagger \hat{d}_{-k} + \hat{c}_{+k}^\dagger \hat{c}_{+k} + \hat{c}_{-k}^\dagger \hat{c}_{-k}] + \sqrt{2}\xi \int_0^\infty dk [\hat{a}_+^\dagger \hat{d}_{+k} + \hat{a}_-^\dagger \hat{d}_{-k} + h.c.]. \quad (7)$$

We see that the cavity modes are decoupled with the fiber mode \hat{c}_{+k} and \hat{c}_{-k} . Choosing parameters $\omega_m = 2J$, we can rewrite Eqs.(5) in the frame rotating at $U = \exp\{-it[-2J(\hat{a}_+^\dagger \hat{a}_+ + \int_0^\infty \Delta_k dk \hat{d}_{+k}^\dagger \hat{d}_{+k}) + \omega_m(\hat{b}_+^\dagger \hat{b}_+ + \hat{b}_-^\dagger \hat{b}_-)]\}$ and neglect the terms rapidly oscillating terms, the Hamiltonian

$$\hat{H}_s = \Delta_-(\hat{a}_+^\dagger \hat{a}_+ + \hat{a}_-^\dagger \hat{a}_-) + \varepsilon_-(\hat{a}_-^\dagger + \hat{a}_-) + \frac{g}{\sqrt{2}}(\hat{a}_+^\dagger \hat{a}_- \hat{b}_-^\dagger + \hat{a}_+ \hat{a}_-^\dagger \hat{b}_-), \quad (8)$$

and

$$\hat{H}_f = \int_0^\infty dk [\Delta_{kJ} \hat{d}_{+k}^\dagger \hat{d}_{+k} + \Delta_k (\hat{d}_{-k}^\dagger \hat{d}_{-k} + \hat{c}_{+k}^\dagger \hat{c}_{+k} + \hat{c}_{-k}^\dagger \hat{c}_{-k})] + \sqrt{2}\xi \int_0^\infty dk [\hat{a}_+^\dagger \hat{d}_{+k} + \hat{a}_-^\dagger \hat{d}_{-k} + h.c.]. \quad (9)$$

where $\Delta_{kJ} = \Delta_k + 2J$. The Hamiltonian Eq.(8) indicate the three-body interaction between cavities and the oscillator, which is exact the same with Ref.³⁴ where the nonlinearity has been analyzed. In a single cavity optomechanical system, the effective photon-photon interactions g^2/ω_m is suppressed by the condition that the mechanical frequency is much larger than the coupling g , i.e., $\omega_m \gg g$, while the three-body interaction (8) has its advantage³⁴ that photons in the two optical modes can be resonantly exchanged by absorbing or emitting a phonon via three-mode mixing; therefore, the restraint $\omega_m \gg g$ can be overcome. Since our system can be simplified as³⁴, one can see that the nonlinearity should be exist and does not restrict by the condition $\omega_m \gg g$. Most importantly, the Hamiltonian Eq.(8) and Eq.(9) exhibit clearly the conversion between the quasi-mode between \hat{a}_+ and \hat{a}_- under the witness of \hat{b}_- so that we can realize the exchange between \hat{d}_{+k} and \hat{d}_{-k} . Therefore, with the interaction, we can potentially realize four ports router.

Photon Blockade

Now we first investigate the nonlinearity of the photons within the cavity and temporally forget the coupling with the fibre. The dynamics of the system obeys the master equation

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}'_{cav} + \hat{H}'_{om}, \hat{\rho}] + \sum_{i=1,2} (\hat{L}_{\hat{a}_i} + \hat{D}_{\hat{b}_i}) \hat{\rho} \quad (10)$$

where $\hat{L}_{\hat{a}_i} = \kappa(2\hat{a}_i \cdot \hat{a}_i^\dagger - \hat{a}_i^\dagger \hat{a}_i \cdot \dots \hat{a}_i^\dagger \hat{a}_i)$, $\hat{D}_{\hat{b}_i} = \gamma_m(n_{thm} + 1)(2\hat{b}_i \cdot \hat{b}_i^\dagger - \hat{b}_i^\dagger \hat{b}_i \cdot \dots \hat{b}_i^\dagger \hat{b}_i) + \gamma_m(2\hat{b}_i^\dagger \cdot \hat{b}_i - \hat{b}_i \hat{b}_i^\dagger \cdot \dots \hat{b}_i \hat{b}_i^\dagger)$, $i = 1, 2$. To characterize the nonlinearity of optical modes, we employ the equal-time second-order correlation functions

$$g_{ij}^{(2)}(0) = \frac{\langle \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i \rangle}{\langle \hat{a}_i^\dagger \hat{a}_i \rangle \langle \hat{a}_j^\dagger \hat{a}_j \rangle}. \quad (11)$$

For $i = j$, the function $g_{ii}^{(2)}(0)$ [$g_{jj}^{(2)}(0)$] denotes the self-correlation, and $g_{ij}^{(2)}(0)$ ($i \neq j$) express the cross-correlation. If the correlation function $g_{ij}^{(2)}(0) < 1$ we say the photon anti-bunching, and the limit $g_i^{(2)}(0) = 0$ corresponds to the thorough photon blockade effect, which means that only one photon can exist, and the another photon will be blocked.

Now, we show the nonlinearity by comparing the numerically solution of the master equation Eq.(10) with that $\hat{H}'_{cav} + \hat{H}'_{om}$ are substituted with effective Hamiltonian Eq.(8) where the subscripts $i = 1, 2$ for the superoperators $\hat{L}_{\hat{a}_i}$ and $\hat{D}_{\hat{b}_i}$ are easily changed to $i = -, +$ because we assume the two cavity modes as well as mechanical modes with equal decay rate respectively. As shown in Fig.2, we see that the solution of master equation with the effective Hamiltonian coincides with that of master equation with original Hamiltonian in most of region except some unstable date resulted from the cut-off error, which show that the effective Hamiltonian method is reliable. We will employ the effective Hamiltonian Eq.(8) in the calculation of the photon router procession. More importantly, we observe that $g_{ij}^{(2)}(0)$ ($i, j = -, +$) achieves their minimum values around $\Delta_- = \pm \frac{g}{\sqrt{2}}$, which means that the system can suppress the simultaneous two-photon creations in any of the mode \hat{a}_- and \hat{a}_+ , especially the cross mode between \hat{a}_- and \hat{a}_+ . That is to say, in the coupled two cavity optomechanical system, there is most possibly only one photon existence. Thus, the property can be potentially used as a single photon router if we can control it. The photon-blockade is resulted from three-body interactions that lead to destructive interference of optical modes. The conclusion is also obtained in³⁴ where the destructive interference is analyzed with eigenstate of the Hamiltonian Eq.(8). The three-body interaction is still dependent on the coupling g see Eq.(8), therefore the strong coupling strength is still welcome. But the nonlinearity is not proportional to $\frac{g^2}{\omega_m}$, which means that the nonlinearity is not limited by the condition $g \ll \omega_m$.

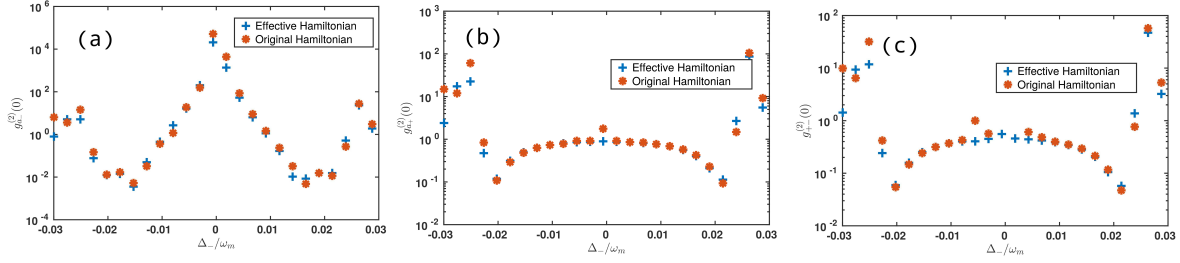


Figure 2. (a) Plot a relation between correlation function $g^{(2)}(0)$ of a_- and detuning Δ_- , blue dot for solve master equation with effective Hamiltonian red dot for original Hamiltonian. (b) Correction $g^{(2)}(0)$ of a_+ as function of Δ_- . (c) Cross correlation function $g_{-+}^{(2)}(0)$ versus detuning Δ_- . Other parameters are $\omega_{m1} = \omega_{m2} = \omega_m$, $J = 2\omega_m$, $g = 0.03\omega_m$, $\kappa = 10^{-3}\omega_m$, $\gamma_m = \kappa/200$, $\varepsilon_1 = 1.1 \times 10^{-4}\omega_m$, $\varepsilon_2 = -\varepsilon_1$

single-photon router

Quantum router is a hinge device for large-scale network communications. How to design quantum router arouse a lot of interests^{1-6,9}. To satisfy the requirements of quantum information, a suitable quantum router should be worked at single-photon state. Photon blockade effect is a effective method to realize the single-photon router.

As we have shown in the Fig.2, there is a good photon blockade phenomenon in this optomechanical system. We can reasonably assume that the device is only allow a single photon transport. Therefore we will only consider a single excitation in the system. Now, we employ the two coupled optomechanical cavities to couple to two waveguide (CRW) shown in Fig.1b. In order to employ the quasi-mode, we introduce medium as phase shifter and beam splitters to generate the quasi-mode. One can easy deduce that the four outputs will satisfy the relation Eq.(6). We now calculate the photon number of the four ports. Under the Hamiltonian Eq.(8) and (9), the bases is denoted as $|n_-, n_+, n_b, n_{\hat{d}_-}, n_{\hat{d}_+}\rangle$, thus we can write the wave function with only a single excitation as

$$|\Psi(t)\rangle = \alpha_- |1, 0, 0, 0, 0\rangle + \alpha_+ |0, 1, 1, 0, 0\rangle + \int_0^\infty dk [\mu_k |0, 0, 0, 1_k, 0\rangle + \eta_k |0, 0, 1, 0, 1_k\rangle], \quad (12)$$

In terms of the left- and right-propagation modes, if we assume a photon packet is incident onto the cavity from the port r_{-k} . The wave function obey Schrodinger equation with Hamiltonian $\hat{H} = \hat{H}_s + \hat{H}_f$. In the long-time limit, we can find the solution of wave function

$$|\Psi(t \rightarrow \infty)\rangle = \int_0^\infty dk [\mu_k(0) e^{-i\Delta_k t} (r_{-k} \hat{r}_{-k} + l_{-k} \hat{l}_{-k}) + \mu'_k(0) e^{-i\Delta_k t} (r_{+k} \hat{r}_{+k} + l_{+k} \hat{l}_{+k})] |0\rangle$$

The details of calculation can be found in part methods. Therefore, the four output photon number are obtained as

$$N_{r_-}^{(out)} = \frac{\pi |G_1|^2}{\varepsilon} - 2\pi |G_1|^2 \gamma^2 \left[\frac{1}{\varepsilon} \frac{\gamma^2 + g^2 + (\delta' + \varepsilon)^2}{\mathcal{F}_{++} \mathcal{F}_{+-} \mathcal{F}_{-+} \mathcal{F}_{--}} + \frac{\sqrt{2}}{4g\gamma} \left(\frac{1}{g/\sqrt{2} + i\gamma} \frac{3g^2/2 - i\sqrt{2}g\gamma}{\mathcal{F}_{++}^* \mathcal{F}_{+-}} + \frac{1}{g/\sqrt{2} - i\gamma} \frac{3g^2/2 + i\sqrt{2}g\gamma}{\mathcal{F}_{-+}^* \mathcal{F}_{--}} \right) \right] \quad (13)$$

$$N_{l_-}^{(out)} = 2\pi |G_1|^2 \gamma^2 \left[\frac{1}{\varepsilon} \frac{\gamma^2 + (\delta' + \varepsilon)^2}{\mathcal{F}_{++} \mathcal{F}_{+-} \mathcal{F}_{-+} \mathcal{F}_{--}} + \frac{\sqrt{2}}{4g\gamma} \left(\frac{1}{g/\sqrt{2} + i\gamma} \frac{g^2/2 - i\sqrt{2}g\gamma}{\mathcal{F}_{++}^* \mathcal{F}_{+-}} + \frac{1}{g/\sqrt{2} - i\gamma} \frac{g^2/2 + i\sqrt{2}g\gamma}{\mathcal{F}_{-+}^* \mathcal{F}_{--}} \right) \right] \quad (14)$$

$$N_{r_+}^{(out)} = \pi |G_1|^2 g^2 \gamma^2 \left[\frac{1}{\varepsilon} \frac{1}{\mathcal{F}_{++} \mathcal{F}_{+-} \mathcal{F}_{-+} \mathcal{F}_{--}} + \frac{\sqrt{2}}{4g\gamma} \left(\frac{1}{g/\sqrt{2} + i\gamma} \frac{1}{\mathcal{F}_{++}^* \mathcal{F}_{+-}} + \frac{1}{g/\sqrt{2} - i\gamma} \frac{1}{\mathcal{F}_{-+}^* \mathcal{F}_{--}} \right) \right] \quad (15)$$

$$N_{l_+}^{(out)} = N_{r_+}^{(out)} \quad (16)$$

with $\mathcal{F}_{\pm\pm} = \delta' \pm g/\sqrt{2} \pm \gamma + i\varepsilon$, where $\delta' = \delta - \Delta_-$. We can clearly see that if $g = 0$, $N_{l_+}^{(out)} = N_{r_+}^{(out)} = 0$, and $N_{r_-}^{(out)}(N_{l_-}^{(out)}) \neq 0$, which means that without the mechanical oscillator we only have two-port router, and the optomechanical coupling is necessary for us to realize multi-port router.

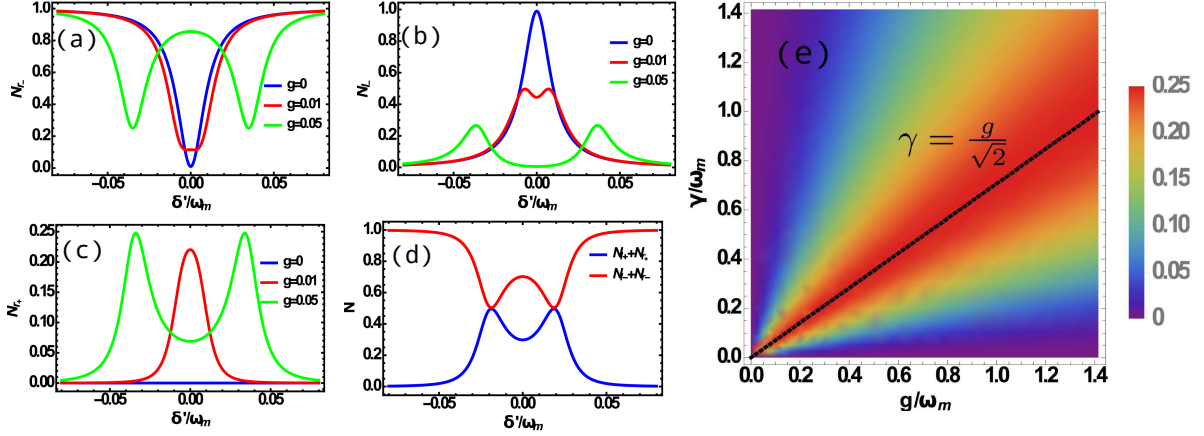


Figure 3. (a),(b),(c) Photon number $N_{r_-}, N_{l_-}, N_{r_+}$ as function of δ' for several values of g where $\gamma = 0.01$. (d) $N_{l_+} + N_{r_+}$ and $N_{l_-} + N_{r_-}$ as function of δ' with $g = 0.04$. It is naturally satisfied normalized condition $N_{l_+} + N_{r_+} + N_{l_-} + N_{r_-} = 1$. (e) Photon number $N_{l_+}(N_{r_+})$ versus γ and g when $\delta' = 0$. And $\varepsilon = 0.0001$ all the parameters were normalized by ω_m .

We plot the output photon number of the four ports as a function of δ' for several values of g in Fig.3(a,b,c). If $g = 0$ (means without the coupling of radiation pressure), when $\delta' = 0$ ($\delta = \Delta_-$ denotes that the input photon is on resonant with the cavity fields), the single photon will almost transmit into the left port \hat{l}_k which was equivalent to a common cavity waveguide coupled system which present a perfect reflection at resonance region. With the increasing of δ' , the photon will be partially transmitted and partially be reflected, but they are still of one peak (valley). Attributing to three-body interactions N_{r_+}, N_{l_+} occurred when $g > 0$. However, increasing the values of g , for example $g = 0.05\omega_m$, the single peak (valley) is split into two peaks (valleys) because the movable mirror participates the three-body interaction so that we can see the symmetry peaks (valleys). Most importantly, the one port input signal can be distributed into four ports see Fig.3(a),(b) and (c), while for $g = 0$, we can receive only two ports signals N_{l_-} and N_{r_-} . This result has been explained above. Therefore, with the assistant of the two coupled cavity optomechanical system, we can realize multi-port router. We parcel the four-port output into two parts $N_{l_+} + N_{r_+}, N_{l_-} + N_{r_-}$ because they denote the difference whether the optomechanical coupling is included or not. Though we can transport the photon via the optomechanical coupling, the probability of transportation $N_{l_+} + N_{r_+}$ is still less than $N_{l_-} + N_{r_-}$ under the group of the parameters. In order to optimized $N_{r_+}(N_{l_+})$, we plot optimized $N_{r_+}(N_{l_+})$ as function of the parameters g and γ shown in Fig.3(e). We observe that when there is an optimized value $N_{r_+}(N_{l_+})$ along the line $\gamma = \frac{g}{\sqrt{2}}$, which exhibit that the balance between cavity waveguide coupling and optomechanical interaction is helpful to the multi-port router procession.

Conclusion

We put forward a scheme to realize multi-port router using two coupled cavity optomechanical system. We first demonstrate that our system with the Hamiltonian Eqs.(8) can be effectively equal to the three-body interaction between cavities and the oscillator which has been shown in³⁴. The nonlinearity in the three-mode mixing is not proportion to g^2/ω_m and can overcome the restraint $\omega_m \gg g$. We also numerically show the nonlinearity and correction of the effective interaction. By coupling the two coupled cavity optomechanical system to waveguide, we calculate the output photon number of the multi-port router. Our results show that the presented system can work as multi-port router under the witness of the optomechanical coupling. Since the two coupled optomechanical cavity is similar with the experiment³⁵ where the optomechanical coupling is ignored. If the the optomechanical coupling is strong enough, our scheme should be realizable.

METHODS

router

Now we solve the Schrodinger equation of this system with Hamiltonian $\hat{H} = \hat{H}_s + \hat{H}_f$ and wave function Eq.(12).

$$\begin{aligned}\dot{\alpha}_- &= -i[\Delta_- \alpha_- + \frac{g}{\sqrt{2}} \alpha_+ + \sqrt{2}\xi \int_0^\infty dk \mu_k], \\ \dot{\alpha}_+ &= -i[\Delta_- \alpha_+ + \frac{g}{\sqrt{2}} \alpha_- + \sqrt{2}\xi \int_0^\infty dk \eta_k], \\ \dot{\mu}_k &= -i[\Delta_k \mu_k + \sqrt{2}\xi \alpha_-], \\ \dot{\eta}_k &= -i[\Delta_{kJ} \eta_k + \sqrt{2}\xi \alpha_+].\end{aligned}\tag{17}$$

We assume that initially the cavity is in the vacuum state, and a single photon with the waveguide, i.e., $|0, 0, 0, 1_k, 0\rangle$ is prepared in a wave packet with a Lorentzian spectrum, the initial condition reads $\mu_k(0) = \frac{G_1}{\Delta_k - \delta + i\epsilon}$. Using Laplace transformation, the differential equations Eq. (17) become

$$\begin{aligned}s\tilde{\alpha}_- &= -i[\Delta_- \tilde{\alpha}_- + \frac{g}{\sqrt{2}} \tilde{\alpha}_+ + \sqrt{2}\xi \int_0^\infty dk \tilde{\mu}_k], \\ s\tilde{\alpha}_+ &= -i[\Delta_- \tilde{\alpha}_+ + \frac{g}{\sqrt{2}} \tilde{\alpha}_- + \sqrt{2}\xi \int_0^\infty dk \tilde{\eta}_k], \\ s\tilde{\mu}_k &= -i[\Delta_k \tilde{\mu}_k + \sqrt{2}\xi \tilde{\alpha}_-] + \mu_k(0), \\ s\tilde{\eta}_k &= -i[\Delta_{kJ} \tilde{\eta}_k + \sqrt{2}\xi \tilde{\alpha}_+],\end{aligned}\tag{18}$$

where $\gamma_j = 2\pi\xi_j^2$ denoting the cavities loss into the waveguide. If there is no the other decay except the exchange between the cavities and the waveguide, γ_j will be equal to the decay rate of the cavity which we have mentioned in Fig.2. For simplicity, we consider $\xi_1 = \xi_2$ so that $\gamma = \gamma_1 = \gamma_2$. In the long-time limit, the coefficients $\mu_k(\infty)$ and $\eta_k(\infty)$ are obtained after inverse Laplace transformation.

$$\begin{aligned}\mu_k(\infty) &= \frac{2(\gamma^2 + \tilde{\Delta}_k^2) - g^2}{2(\tilde{\Delta}_k + i\gamma)^2 - g^2} \mu_k(0) e^{-i\Delta_k t}, \\ \eta_k(\infty) &= -\frac{2\sqrt{2}ig\gamma}{2(\tilde{\Delta}_{kJ} + i\gamma)^2 - g^2} \mu'_k(0) e^{-i\Delta_{kJ} t}.\end{aligned}\tag{19}$$

In terms of the left- and right-propagation modes, if we assume a photon packet is incident onto the cavity from the port r_{-k} , then the initial state can be written as

$$|\Psi(0)\rangle = \int_0^\infty dk \mu_k(0) \hat{r}_{-k} |\emptyset\rangle = \frac{1}{\sqrt{2}} \int_0^\infty dk \mu_k(0) (\hat{d}_{-k} + \hat{c}_{-k}) |\emptyset\rangle,\tag{20}$$

which means that the single photon input from the port r_{-k} can be considered as a superposition between a quasiparticle \hat{d}_{-k} and a quasiparticle \hat{c}_{-k} . In the long-time limit, the wave function becomes under the Hamiltonian Eq.(8) and Eq.(9)

$$|\Psi(t \rightarrow \infty)\rangle = \int_0^\infty dk [\mu_k(0) e^{-i\Delta_k t} (r_{-k} \hat{r}_{-k} + l_{-k} \hat{l}_{-k}) + \mu'_k(0) e^{-i\Delta_{kJ} t} (r_{+k} \hat{r}_{+k} + l_{+k} \hat{l}_{+k})] |\emptyset\rangle$$

, where the first bracket with the factor $e^{-i\Delta_k t}$ can survive without Hamiltonian Eq.(8), while the second bracket with the factor $e^{-i\Delta_{kJ} t}$ survive only under the condition Eq.(8) existence. In other words, the photon on the ports r_{-k} and l_{-k} can be detected even without the mechanical mode, however, if we would like to obtain photon on the port r_{+k} and l_{+k} , the coupling between the mechanical mode and cavity field is necessary. Then we obtain

$$\begin{aligned}r_{-k} &= \sqrt{2} \frac{(\gamma^2 + \tilde{\Delta}_k^2) + (\tilde{\Delta}_k + i\gamma)^2 - g^2}{2(\tilde{\Delta}_k + i\gamma)^2 - g^2}, \\ l_{-k} &= \sqrt{2} \frac{\gamma^2 + \tilde{\Delta}_k^2 - (\tilde{\Delta}_k + i\gamma)^2}{2(\tilde{\Delta}_k + i\gamma)^2 - g^2}, \\ l_{+k} &= -\frac{2ig\gamma}{2(\Delta_k + i\gamma)^2 - g^2}, \\ r_{+k} &= l_{+k},\end{aligned}\tag{21}$$

and the output photon number in Eq.(14),(15),(16).

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Author contributions statement

X.L. and L.Z. designed the research, X.L. did the analytic calculations, W.Z.Z. provided numerical and prepared figures, L.Z. revised the manuscript and provided overall theoretical support.

Additional information

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